

Balanced right/left-handed mixtures of quasi-planar chiral inclusions

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Abstract

Some novel quasi-planar chiral inclusions, feasible from standard photo-etching techniques, are proposed. It is shown that such inclusions can be designed in order to present balanced electric, magnetic and magneto-electric polarizabilities. Using these inclusions, random and periodic bi-isotropic artificial metamaterials exhibiting a balanced positive/negative refractive index can be build up. These metamaterials would exhibit reasonable bandwidths and excellent matching to free space

1. Introduction

In spite of some proposal in such direction [1] – [9], the development of bulk isotropic metamaterials exhibiting negative refractive index (NRI) and good matching to free space is still a challenging issue. A promising approach to this problem could take advantage of the simultaneous electric and magnetic polarizability of chiral inclusions in order to obtain a mixture with simultaneously negative ε and μ . As far as we know, the first proposal in such direction was made in [10], and further developed in [11]. In these works racemic and chiral mixtures of chiral metallic inclusions were proposed as a practical way of designing left-handed metamaterials. Other proposals taking advantage of chirality for negative refractive artificial media design have also been made (see [12] and [13], for instance). In this contribution we will further develop this approach presenting some novel NRI bi-isotropic metamaterial designs that exhibit reasonable bandwidths and excellent matching to free space.

2. Balanced positive/negative refractive index (BPNRI) metamaterials

BPNRI metamaterials are defined as bi-isotropic media exhibiting balanced electric, χ_e , and magnetic χ_m susceptibilities, that is

$$\chi_e(\omega) = \chi_m(\omega), \quad (1)$$

over a wide bandwidth which includes both positive and negative values of $\varepsilon_r = 1 + \chi_e$ and $\mu_r = 1 + \chi_m$. Such media are characterized by the following properties:

- Wide NRI pass-band for the positive and/or the negative circularly polarized TEM eigenwaves, defined by the condition $\sqrt{\varepsilon_r \mu_r} \pm \kappa < 0$ [14] where the negative sign for the square root must be chosen if ε and μ are both negative.

- No forbidden bands, as it is deduced from the dispersion equation $k^\pm = k_0 (\sqrt{\mu_r \varepsilon_r} \pm \kappa)$ with $k_0 = \omega \sqrt{\epsilon_0 \mu_0}$.
- Transition between the NRI and PRI pass-bands through a zero phase velocity point (a behavior similar to that previously reported for some transmission line metamaterials [15]).
- Good matching (perfect matching for paraxial rays) to free space, due to the matching of TEM impedances $\eta = \sqrt{\mu/\varepsilon} = \eta_0$.

In the following we will develop a design with $|\kappa| \sim \chi_e, \chi_m$. In such case only one of the TEM circularly polarized eigenwaves will exhibit negative refraction and the NRI pass-band is defined by the condition $\chi_e = \chi_m < -0.5$ [16]

3. Balanced quasi-planar chiral inclusions

Figure 1 shows two quasi-planar chiral inclusions suitable for balanced metamaterial design: the chiral split ring resonator (Ch-SRR) and the chiral spiral resonator (Ch-SR). They are the broadside-coupled versions of the spiral resonator [17] and the NB-SRR [18] previously proposed by some of the authors. In both cases the frequency of resonance can be obtained from an LC circuit model: $\omega_0 = 1/\sqrt{LC}$ where L and C are the effective inductance and capacitance of the inclusion (analytical expressions can be found in [19]). The polarizabilities can be obtained following the standard technique developed in [19] and in [17] – [18]. This calculation gives:

$$\alpha_{zz}^{mm} = \alpha_0^{mm} \frac{\omega^2}{\omega_0^2 - \omega^2 + j\omega R/L}; \quad \alpha_0^{mm} = \frac{\pi^2 r^4}{L} \quad (2)$$

$$\alpha_{zz}^{em} = \pm \alpha_0^{em} \left(\frac{\omega_0}{\omega} \right) \frac{\omega^2}{\omega_0^2 - \omega^2 + j\omega R/L}; \quad \alpha_0^{em} = j \frac{n\pi r^2 t}{\omega_0 L} \left(\frac{C_0}{C} \right) \quad (3)$$

$$\alpha_{zz}^{ee} = \alpha_0^{ee} \left(\frac{\omega_0}{\omega} \right)^2 \frac{\omega^2}{\omega_0^2 - \omega^2 + j\omega R/L}; \quad \alpha_0^{ee} = \frac{(nt)^2}{\omega_0^2 L} \left(\frac{C_0}{C} \right)^2 \quad (4)$$

$$\alpha_{xx}^{ee} = \alpha_{yy}^{ee} = \alpha_0; \quad \alpha_0 = \varepsilon_0 \frac{16}{3} r_{ext}, \quad (5)$$

where $n = 1$ for the Ch-SR and $n = 2$ for the Ch-SRR (see [19] for more details on notation). From (2) – (4) it follows that

$$\alpha_{zz}^{mm} \alpha_{zz}^{ee} + (\alpha_{zz}^{em})^2 = 0, \quad (6)$$

which is a general property arising from the LC nature of the model [11]. In order to obtain a balanced design we will further impose

$$\alpha_0^{ee} = \mu_0 \varepsilon_0 \alpha_0^{mm}, \quad (7)$$

which is satisfied provided that

$$t\lambda_0 = \frac{2}{n} \frac{C}{C_0} (\pi r)^2 \quad (8)$$

where λ_0 is the wavelength at resonance. Incidentally, since the frequency of resonance of the Ch-SRR is twice that of the Ch-SR, (8) provides exactly the same geometrical parameters for both inclusions. The accuracy of this expression for giving a balanced design has been shown in [16] by electromagnetic simulation.

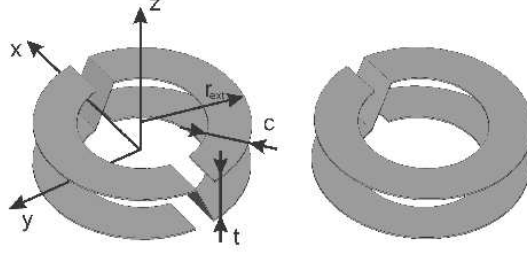


Figure 1: Two quasi-planar chiral inclusions, the chiral SRR (Ch-SRR, left) and the chiral spiral resonator (Ch-SR, right). For practical designs metallic rings can be photo-etched on both sides of a dielectric board and connected through a via-hole.

4. Isotropic arrangement

In order to obtain a BPNRI metamaterial the inclusions shown in Fig.1 must be properly arranged. The main advantage of the Ch-SR is its small electrical size (balanced designs with sizes of about $\lambda_0/13$ can be obtained [16]). However, the Ch-SR has not enough symmetry to allow for the design of an isotropic cubic resonator, suitable for a periodic isotropic design [6]. This condition is fulfilled by the Ch-SRR which allows for the design of cubic arrangements satisfying the cubic T group of symmetry (in Schoenflies notation), which is enough to guarantee an isotropic metamaterial design [6]. Therefore, Ch-SRs are suitable for the design of random BPNRI media and the Ch-SRR is appropriate for the design of periodic BPNRI media. In both cases a Lorentz homogenization procedure provides the following relation between the macroscopic fields and volume polarizations

$$M = N \left\{ \mu_0 \hat{\alpha}^{mm} \left(H + \frac{M}{3} \right) - \hat{\alpha}^{em} \left(E + \frac{P}{3\epsilon_0} \right) \right\} \quad (9)$$

$$P = N \left\{ \hat{\alpha}^{ee} \left(E + \frac{P}{3\epsilon_0} \right) + \mu_0 \hat{\alpha}^{em} \left(H + \frac{M}{3} \right) \right\}, \quad (10)$$

where N is the number of particles per unit volume, and $\hat{\alpha}^{mm}$, $\hat{\alpha}^{ee}$, $\hat{\alpha}^{em}$ are some average polarizabilities given by $\hat{\alpha}^{mm} = \alpha_{zz}^{mm}/3$, $\hat{\alpha}^{em} = \alpha_{zz}^{em}/3$ and $\hat{\alpha}^{ee} = (\alpha_{xx}^{ee} + \alpha_{yy}^{ee} + \alpha_{zz}^{ee})/3$

For a balanced design of the inclusions, taking (8) into account, we finally find

$$\chi_e = \frac{N}{3\Lambda} \left\{ \frac{2\alpha_0}{\epsilon_0} \left(\frac{\omega_0^2}{\omega^2} - 1 \right) + \mu_0 \alpha_0^{mm} \left(\frac{\omega_0^2}{\omega^2} - \frac{2N\alpha_0}{9\epsilon_0} \right) \right\} \quad (11)$$

$$\chi_m = \frac{N\mu_0\alpha_0^{mm}}{3\Lambda} \left(1 - \frac{2N\alpha_0}{9\epsilon_0} \right) \quad (12)$$

$$\kappa = \pm \frac{\omega_0}{\omega} \frac{N\mu_0\alpha_0^{mm}}{3\Lambda} \quad (13)$$

where

$$\Lambda = K \left\{ \frac{\omega_0^2}{\omega^2} - 1 + \frac{N\mu_0\alpha_0^{mm}}{9} \left(1 + \frac{\omega_0^2}{K\omega^2} \right) + j \frac{R}{\omega L} \right\} ; \quad K = \left(1 - \frac{2N\alpha_0}{9\epsilon_0} \right) \quad (14)$$

These expressions satisfy (1) in the limit $\omega_0/\omega \rightarrow 1$. Since most resonant metamaterials have a moderate bandwidth ($\sim 10\%$ or less), this condition is approximately fulfilled inside the left-handed pass-band.

Figure 2 shows the propagation constants and impedances, $k^\pm = k_0 (\sqrt{\mu_r \epsilon_r} \pm \kappa)$ and $\eta = \sqrt{\epsilon/\mu}$, computed from (11) – (13) for the TEM eigenwaves in a compact arrangement of balanced

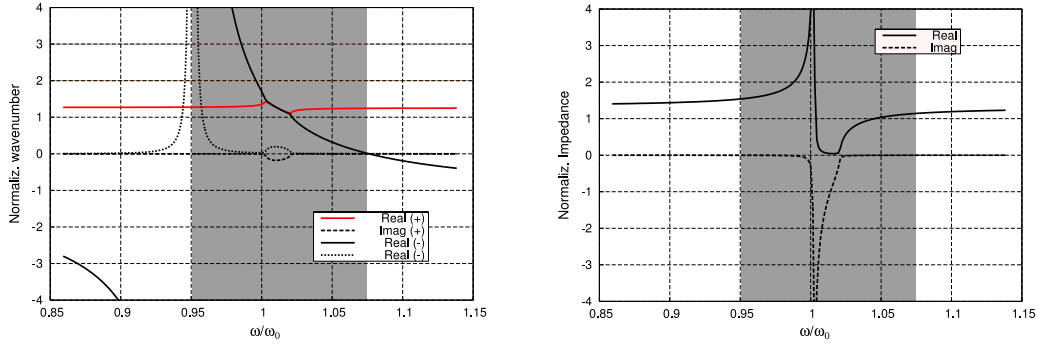


Figure 2: Plots of k^\pm/k_0 ($k_0 = \omega\sqrt{\epsilon_0\mu_0}$) (left) and η/η_0 against ω/ω_0 (right) for a cubic *fcc* lattice of balanced Ch-SRs with $c/r_{ext} = 0.2$, and $t/r_{ext} = 0.306$. Substrate is foam with $\epsilon = \epsilon_0$ and N is calculated in the text. The NRI pass-band is marked in the figures.

Ch-SRs randomly oriented. N corresponds to a cubic *fcc* lattice of balls with radius $a = 1.1r_{ext}$ which contains the Ch-SRs, i.e. $N = 0.74 \left(\frac{4}{3}\pi(1.1r_{ext})^3 \right)^{-1}$. The remaining parameters are given in the caption. As it can be seen, a BPNRI behavior is quite approximately obtained.

5. Conclusion

Along this paper a fully analytical method for designing bi-isotropic balanced positive/negative refractive index metamaterials has been presented. Some new quasi-planar chiral inclusions have been proposed for this purpose. These inclusions can be easily manufactured by standard photo-etching techniques and the condition for balanced design can be expressed in a quite simple way (8). Hopefully, the interesting properties of bi-isotropic BPNRI metamaterials, such as the smooth transition between the NRI and the PRI pass-bands and the excellent matching to free space, could find application in focusing devices and other applications.

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